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The effects of pump phase modulation on noise characteristics of fiber optical parametric amplifiers

M. Gao[a], C. Jiang[b], W. Hu, J. Wang, and H. Ren

State Key Laboratory of Advanced Optical Communication Systems & Networks Shanghai Jiao Tong University, Shanghai 200240, P.R. China

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Abstract. This paper investigates the effects of pump phase modulation (PM) on the noise characteristics of fiber optical parametric amplifiers (FOPAs) theoretically. The results show that the effect of the pump PM on the noise of FOPAs can be diminished by adjusting the structural parameters of amplifiers properly. The parameters of the pump phase modulation are optimized with several methods to reduce the noise of FOPAs.

PACS. 42.65.Yj Optical parametric oscillators and amplifiers – 42.65.-k Nonlinear optics – 42.65.Ky Frequency conversion; harmonic generation, including higher-order harmonic generation – 42.65.Sf Dynamics of nonlinear optical systems; optical instabilities, optical chaos and complexity, and optical spatio-temporal dynamics

1 Introduction

At present, wavelength-division multiplexing (WDM) is applied increasingly to the optical communication systems to increase the information capacity of each fiber and it requires amplifiers to compensate the loss of fiber and equalize the power of the various channels. Fiber optical parametric amplifiers (FOPAs), which utilize four-wave mixing (FWM) to amplify the signals, could offer a broadband and high-gain amplification at arbitrary wavelengths [1–4].

To achieve a high gain over a relatively wide gain bandwidth, FOPAs require quite high pump power. However, the stimulated Brillouin scattering (SBS) limits the maximum pump power of FOPAs. In most cases, FOPAs are pumped by a phase-modulated or frequency-modulated continuous-wave (CW) to widen the spectral width of the pump wave, which raises the SBS threshold [5–7]. But the pump phase modulation (PM) affects the performance of FOPAs: it may alter the signal gain, broaden the idler spectrum and increase the Q penalties [8–10]. The techniques to synchronously modulate the pump and the signal have been proposed to cancel the spectral spread [11]. However, the problems of the strong gain-modulation caused by the pump PM still exist and result in the fluctuations of signal output power. Thus, the gain modulation caused by the pump PM introduces noise to signal and degrades the performance of FOPAs.

Some measures have been proposed to suppress the effects of the pump PM on the noise of FOPAs, such as using fibers with variations of zero dispersion wavelength λ_0, with very short length or with very low dispersion slope [5]. Overall, they mainly focus on the design of fiber. In the paper, the researches put emphasis on adjusting the structural parameters of amplifiers for noise reduction of FOPAs, such as decreasing the pump power and the length and nonlinear coefficient of fiber. Moreover, several methods are also proposed to optimize the parameters of the pump PM to reduce the noise of FOPAs, such as decreasing the pump phase modulation frequency and modulation index and using multiple sinusoidal signals to modulate the pump.

2 Theoretical model and analysis

The signal gain of FOPAs can be expressed as [12],

$$G_s = 1 + \left[\frac{\gamma P_p}{g}\sinh(gL)\right]^2 \quad (1)$$

where γ is the nonlinear coefficient of fiber, P_p is the pump power, L is fiber length, g is parametric gain coefficient $g^2 = -\Delta\beta(\Delta\beta/4 + \gamma P_p)$, $\Delta\beta$ is the linear propagation-constant mismatch $\Delta\beta = \beta^{(2)}\Omega^2 + 1/12\beta^{(4)}\Omega^4$, Ω is the frequency detuning between the signal and the pump frequency $\Omega = \omega_s - \omega_p$, the $\beta^{(2)}$ is the second-order dispersion related to the third- and fourth-order dispersion

[a] e-mail: gmy@sjtu.edu.cn
[b] e-mail: cjiang@sjtu.edu.cn

($\beta^{(3)}$ and $\beta^{(4)}$) at the zero-dispersion frequency ω_0 of fiber $\beta^{(2)}(\omega_p) = \beta^{(3)}(\omega_0)(\omega_p - \omega_0) + \beta^{(4)}(\omega_0)(\omega_p - \omega_0)^2/2$.

Supposing a sinusoidal wave $\sin(\omega_m t)$ is used to modulate the phase of the pump wave, the phase-modulated φ is expressed as $\varphi(\tau) = m \sin(\omega_m \tau)$, where m is the modulation index and ω_m is the modulation frequency. After the pump phase is modulated, the instantaneous parametric gain coefficient of FOPAs can be given by [9]:

$$g(\tau)^2 = -(\Delta\beta + \delta\beta(\tau))\left[(\Delta\beta + \delta\beta(\tau))/4 + \gamma P_0\right]. \quad (2)$$

Compared with the case without the pump PM, an instantaneous phase mismatch $\delta\beta(\tau)$ caused by the pump PM, appears in the parametric gain coefficient, which is expressed as [9]:

$$\begin{aligned}\delta\beta(\tau) = \beta^{(2)}\varphi_\tau^2 - \beta^{(3)}(\varphi_\tau \Omega^2 + 1/3\varphi_\tau^3) \\ + 1/12\beta^{(4)}(\varphi_\tau^4 + 6\varphi_\tau^2\Omega^2). \quad (3)\end{aligned}$$

Here φ_τ is the first derivative of the pump phase,

$$\varphi_\tau = \partial\varphi/\partial\tau = m\omega_m \cos(\omega_m \tau). \quad (4)$$

Under the condition of small instantaneous phase mismatch, only the linear term of φ_τ is kept and $\delta\beta(\tau)$ is approximated to,

$$\delta\beta(\tau) \approx -\beta^{(3)}\varphi_\tau \Omega^2. \quad (5)$$

The instantaneous gain of FOPAs can be expressed according to the mean gain $\langle G_s \rangle$ and may be approximated under the condition of small instantaneous phase mismatch

$$\begin{aligned}G_s(\tau) &= 1 + [\gamma P_0/g(\tau)\sinh(g(\tau)L)]^2 \\ &\approx 1 + [\gamma P_0/g\sinh(gL)]^2 \left[1 + \left(\frac{\Delta\beta + 2\gamma P_0}{2g_0^2}\right.\right. \\ &\quad \left.\left. - \frac{aL\sinh(2gL)}{\sinh^2(gL)}\right)\delta\beta(\tau)\right] \\ &\approx \langle G_s \rangle \left[1 + \left(\frac{\Delta\beta + 2\gamma P_0}{2g_0^2} - \frac{aL\sinh(2gL)}{\sinh^2(gL)}\right)\delta\beta(\tau)\right] \quad (6)\end{aligned}$$

where $\delta\beta(\tau) = -\beta^{(3)}\Omega^2 m\omega_m \cos\omega_m\tau$ and

$$a = \frac{1}{4}\frac{\Delta\beta + 2\gamma P_0}{\sqrt{-\Delta\beta(1/4\Delta\beta + \gamma P_0)}} = \frac{1}{4}\frac{\Delta\beta + 2\gamma P_0}{g_0}.$$

The mean square fluctuation of the instantaneous phase mismatch $\delta\beta(\tau)$ can be expressed as:

$$\left\langle \delta\beta(\tau)^2 \right\rangle = 1/2\left(\beta^{(3)}\Omega^2 m\omega_m\right)^2. \quad (7)$$

A variable substitute is made:

$$\Lambda = \frac{\Delta\beta + 2\gamma P_0}{4g_0}\frac{L\sinh(2gL)}{\sinh^2(gL)} - \frac{\Delta\beta + 2\gamma P_0}{2g_0^2}. \quad (8)$$

Substituting equations (7) and (8) into equation (6):

$$G_s = \langle G_s \rangle \left(1 + \Lambda\sqrt{2}\sqrt{\langle \delta\beta(\tau)^2 \rangle}\cos(\omega_m\tau)\right) = \langle G_s \rangle + \delta G_s \quad (9)$$

where

$$\langle G_s \rangle = 1 + [\gamma P_0/g\sinh(gL)]^2$$

and

$$\delta G_s = \langle G_s \rangle \Lambda\sqrt{2}\sqrt{\langle \delta\beta(\tau)^2 \rangle}\cos(\omega_m\tau).$$

The mean square fluctuations in output signal power is obtained,

$$\begin{aligned}\sigma_{s-PM}^2 = \left\langle \delta P_s^2 \right\rangle &= \langle P_s \rangle^2 \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\delta G_s}{\langle G_s \rangle}\right)^2 d\theta \\ &= P_{s0}^2 \frac{1}{2\pi}\int_0^{2\pi}(\delta G_s)^2 d\theta \\ &= P_{s0}^2 \langle G_s \rangle^2 \Lambda^2 \left\langle \delta\beta(\tau)^2 \right\rangle \\ &= 1/2\left(\beta^{(3)}\Omega^2 m\omega_m\right)^2 P_{s0}^2 \langle G_s \rangle^2 \Lambda^2 \quad (10)\end{aligned}$$

where $\theta = \omega_m\tau$. In fact, in order to suppress stimulated Brillouin scattering (SBS), the pump spectrum was broadened by several combined sinusoidal signals [5,6]. In the scenario, the phase-modulated φ is expressed as $\varphi(\tau) = \sum_{i=1}^{n} m_i \sin\omega_{mi}\tau$ and the first derivative of the pump phase φ_τ is given by

$$\varphi_\tau = \partial\varphi/\partial\tau = \sum_{i=1}^{n} m_i\omega_{mi}\cos\omega_{mi}\tau.$$

The mean square fluctuation of the instantaneous phase mismatch $\delta\beta(\tau)$ can be expressed as

$$\left\langle \delta\beta(\tau)^2 \right\rangle = (1/2)(\beta^{(3)}\Omega^2)^2 \sum_{i=1}^{n}(m_i\omega_{mi})^2.$$

The mean square fluctuations in output signal power is given,

$$\begin{aligned}\sigma_{s-PM}^2 &= P_{s0}^2 \langle G \rangle^2 \Lambda^2 \left\langle \delta\beta(\tau)^2 \right\rangle \\ &= (1/2)\left(\beta^{(3)}\Omega^2 P_{s0}\langle G \rangle \Lambda\right)^2 \sum_{i=1}^{n}(m_i\omega_{mi})^2. \quad (11)\end{aligned}$$

The noise figure of FOPAs can be expressed as [13,14]:

$$\begin{aligned}F(dB) &= 10\log\left(\frac{SNR_{in}}{SNR_{out}}\right) \\ &= 10\log\left(\frac{\frac{h\nu\langle n_s \rangle}{h\nu/2}}{\frac{h\nu G_s\langle n_s \rangle}{(h\nu/2)[G_s + (2n_{sp}-1)(G_s-1)]}}\right) \\ &= 10\log\left[1 + (2n_{sp}-1)(1 - 1/G_s)\right]. \quad (12)\end{aligned}$$

In the case of an ideally fully inverted amplifier ($n_{sp} = 1$) and large gain, the noise figure is equal to 3 dB. However, if the variance of the signal intensity caused by the pump

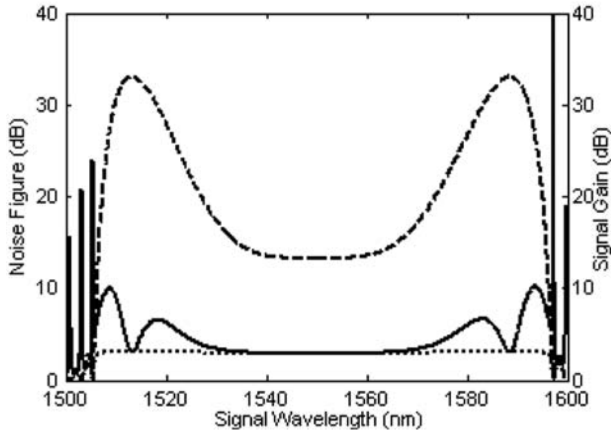

Fig. 1. The curves of ideal noise figure for FOPA (dotted curve), the noise figure including the effects of the pump PM (solid curve) and the signal gain (dashed curve).

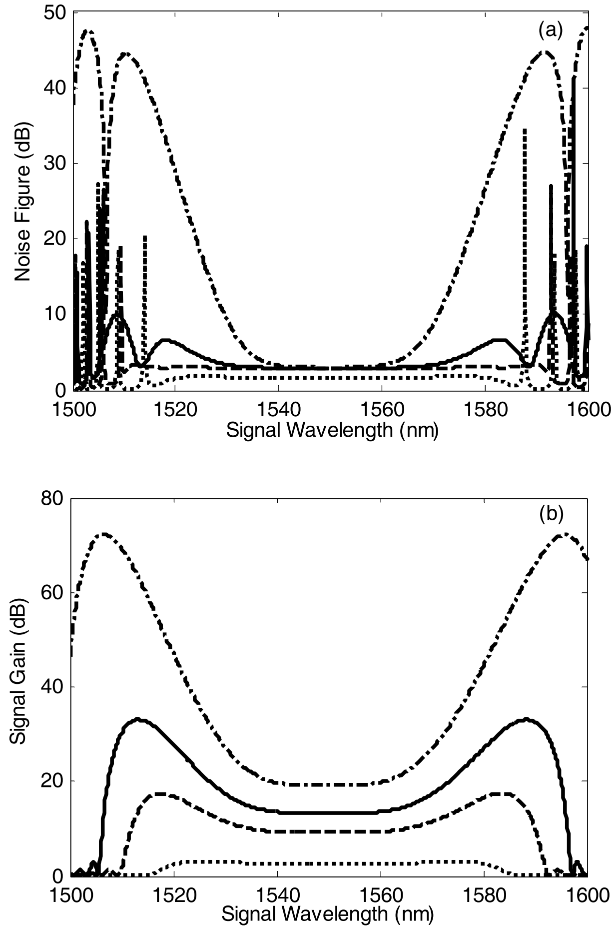

Fig. 2. The curves of noise figure (a) and the corresponding signal gain (b) for FOPA with the pump power of 0.1 W (dotted curve), 0.3 W (dashed curve), 0.5 W (solid curve) and 1 W (dash-dotted curve).

PM is taken into account, equation (12) will be rewritten as [15,16]:

$$F(dB) = 10\log\left(\frac{\frac{h\nu\langle n_s\rangle}{h\nu/2}}{\frac{h\nu G_s\langle n_s\rangle}{(h\nu/2)[G_s+(2n_{sp}-1)(G_s-1)]+S}}\right)$$
$$= 10\log\left[1+(2n_{sp}-1)(1-1/G_s)+S/(G_s h\nu/2)\right] \quad (13)$$

where h is Planck's constant, υ is the optical frequency, n_{sp} is the population inversion factor, G_s is the signal gain and S is the spectral noise density from the pump PM, which can be calculated by $S = \sigma^2_{s-PM}/B_e$ and B_e is the electrical filter bandwidth of 10 GHz.

3 Simulation results and discussion

Supposing the pump phase modulation index m is 1 and the pump phase modulation frequency f_m is 0.2 GHz, simulations were performed to investigate the effect of pump PM on the noise characteristics of FOPAs. In simulations, the third- and fourth-order dispersion of fiber $\beta^{(3)} = 1.2 \times 10^{-40}$ s^3/m and $\beta^{(4)} = -2.85 \times 10^{-55}$ s^4/m, the zero-dispersion wavelength of fiber $\lambda_0 = 1550$ nm, the pump wavelength of FOPA $\lambda_p = 1550$ nm. Moreover, the FOPA has the fiber length L of 0.5 km, the pump power P_p of 0.5 W, the nonlinear coefficient of fiber γ of 18 km^{-1} W^{-1} and the input signal power P_{s0} of 0.1 mW. These fiber parameters come from reference [17]. Figure 1 shows the noise figure and the signal gain as functions of the signal wavelength. In the case of an ideally fully inverted FOPA and large gain, the noise figure is about 3 dB, as shown by dotted curve in Figure 1. When the effects of the pump PM are considered, the noise of FOPA deviates from the ideal noise for the signal wavelength being far from the pump wavelength, as shown by solid curve in Figure 1. And the dashed curve in Figure 1 shows the signal gain of FOPA.

And then the effects of the structural parameters of amplifiers, such as the pump power and the length and nonlinear coefficient of fiber, on the noise characteristics of FOPAs are analyzed, as follows. Supposing that the pump power P_p is 0.1 W, 0.3 W, 0.5 W and 1 W, respectively and the other parameters are same as those used in Figure 1, the curves of the noise figure of FOPA are shown in Figure 2a. It is very clear that with the increase of the pump power, the noise of FOPAs including the effects of the pump PM is also increased. Moreover, an increasing pump power also results in a rise in the signal gain of FOPA, as shown in Figure 2b. In the case of the pump power less than 0.3 W, the noise figure in the usable wavelength range is about 3 dB. However, when the pump power increases to 1 W, the noise of FOPA deviates away from the ideal noise of 3 dB to a high level for the signal wavelength being far away the pump wavelength.

Moreover, simulations were performed to investigate the effects of the fiber length on the noise of FOPAs. The results show that the increase of the fiber length causes the noise to rise. Therefore, a shorter fiber is more suitable for a practical FOPA, which not only provides a broader gain

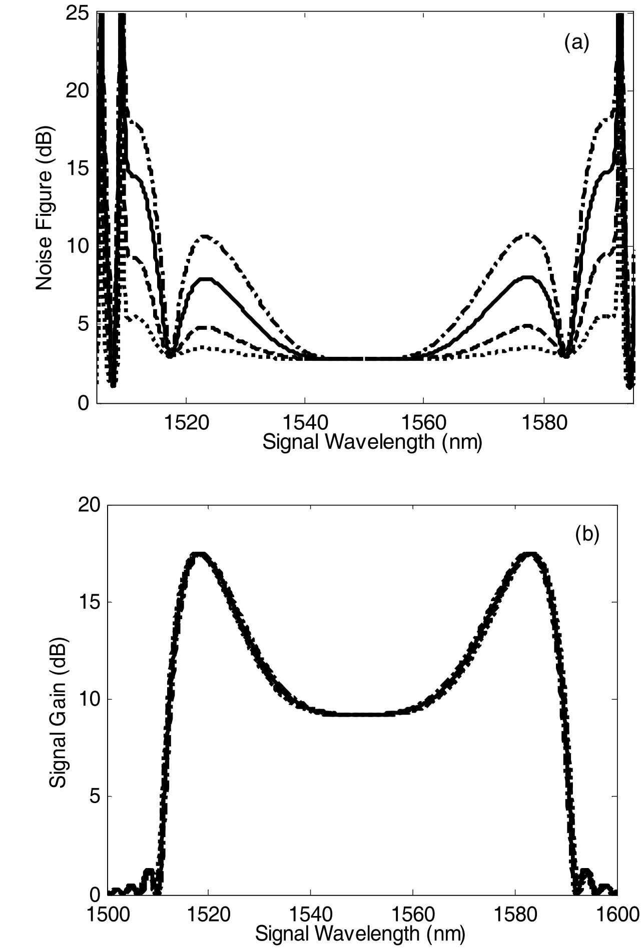

Fig. 3. The curves of noise figure (a) and the corresponding signal gain (b) for FOPA with the pump phase modulation frequency of 0.5 GHz (dotted curve), 1 GHz (dashed curve), 2 GHz (solid curve) and 3 GHz (dash-dotted curve).

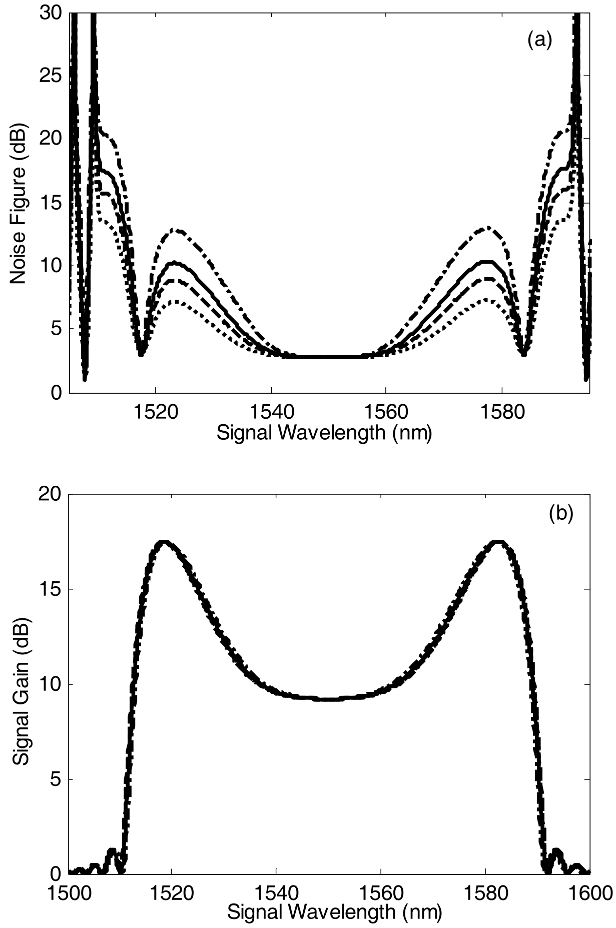

Fig. 4. The curves of noise figure (a) and the corresponding signal gain (b) for FOPA with one (dash-dotted curve), two (solid curve), three (dashed curve) and four sinusoidal signals (dotted curve).

bandwidth and a smaller noise but also decreases the effect of zero-dispersion wavelength's fluctuation. But a FOPA with a shorter fiber has a lower parametric gain. Thus, in order to obtain a higher parametric gain, highly nonlinear fiber and microstructure fiber with a higher nonlinear coefficient can be applied to FOPA [18]. However, the increase of the fiber nonlinear coefficient also deteriorates the noise performance.

The preceding methods focus on adjusting the structural parameters of FOPAs to reduce the effects of the pump PM on noise of FOPAs, however, at the same time they also significantly change the parametric gain of FOPAs, as shown in Figure 2b. Therefore, other methods are proposed to optimize the parameters of the pump PM for noise reduction. Figure 3a shows the curves of noise figure of FOPA with the pump phase modulation frequency f_m of 0.5 GHz, 1 GHz, 2 GHz and 3 GHz, respectively. Assume that the pump power is 0.3 W and the rest of the parameters are same as those used in Figure 1. In Figure 3a, it is very clear that the noise of FOPA reduces with the decrease of the pump phase modulation frequency for the signal wavelength being far away the pump wavelength. However, there are only small changes in signal gain of FOPA with the pump phase modulation frequency, as shown in Figure 3b. For the pump phase modulation frequency of less than 0.5-GHz, the deviation from the ideal noise is minimized in the usable wavelength range and thus the effects of the pump PM on the noise can be neglected. The same results can be achieved by decreasing the pump phase modulation index m. Therefore, for a FOPA with fixed structural parameters, by reducing the pump phase modulation frequency and the modulation index, the effects of the pump PM on the noise can be reduced to a much lower level but the signal gain nearly remains unchanged.

Simulations, in which multiple sinusoidal signals were combined to broaden the pump spectrum for suppressing the SBS, were also performed and the results were compared. In the first simulation a single sinusoidal signal with a modulation frequency of 4 GHz was used to modulate the pump, whose curve of noise figure is shown by dotted curve in Figure 4a. Then multiple sinusoidal signals with same modulation frequency were used in each of the following three simulations: two signals with a modulation frequency of 2 GHz, three with 4/3 GHz and four with

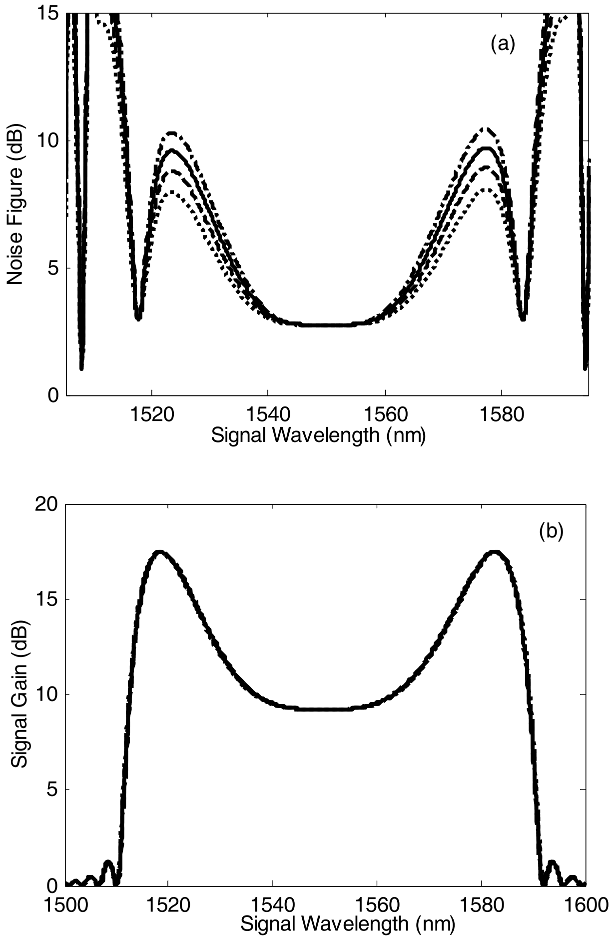

Fig. 5. The curves of noise figure (a) and the corresponding signal gain (b) for FOPA modulated by four sinusoidal signals with modulation frequency of 1, 1, 1 and 1 GHz (dotted curve), 0.3, 0.7, 1.2, 1.8 GHz (dashed curve), 0.1, 0.3, 1.6, 2.0 GHz (solid curve) and 0.1, 0.3, 0.9, 2.7 GHz (dash-dotted curve).

1 GHz. In all simulations, all modulation indexes m are 1 and the rest of the parameters of FOPA are same as those used in Figure 3. From Figure 4a, it can be shown that when the numbers of sinusoidal signals used to modulate the pump were increased, the effects of the pump PM on the noise of FOPA were reduced. At the same time, the signal gain of FOPA has only small deviation, as shown in Figure 4b. Therefore, using more sinusoidal signals to modulate the pump is beneficial for noise reduction. However, it makes the configuration of FOPAs more complicated. Therefore, we should try to balance between broadening pump spectrum to suppress the SBS, reducing the effects of the pump PM on the noise and simplifying the pump configuration according to our purpose.

Furthermore, when the pump is modulated by multiple sinusoidal signals, it is very important to optimize their modulation frequencies. Suppose that four sinusoidal signals with different modulation frequencies are used to modulate the pump and the rest of the parameters of FOPA are same as those used in Figure 3. Simulations were performed to find the optimum combination of signal modulation frequencies and the curves of noise figure were shown in Figure 5a. The results indicate that different combinations of pump modulation frequencies cause different levels of noise. However, when signal wavelength is close to the pump wavelength, the noise figure of FOPA for all modulation signals is close to the ideal one. When the multiple modulation signals have the same frequencies, the noise figure is smaller, as shown by dotted curve in Figure 5a. But their impact on suppressing SBS is worse than those with different modulation frequencies, in which the pump power is distributed to different frequencies, and the SBS threshold increases. For those modulation signals with different modulation frequencies, when the summation of modulation frequencies' square becomes smaller, the effects of the pump PM on the noise of FOPA will be reduced, as shown by dashed curve in Figure 5a. Moreover, the differences of modulation frequencies among signals have only a little influence on the signal gain of FOPA, as shown in Figure 5b.

4 Conclusion

The effects of the pump PM on the noise characteristic of single-pump FOPAs were derived analytically. The studies show that the structural parameters of FOPAs should be optimized to balance the conflicting requirement of high gain and low noise. For a FOPA with fixed structure parameters, the noise can be reduced by measures to improve the pump PM, such as decreasing the pump phase modulation frequency and modulation index, and using multiple sinusoidal signals to modulate the pump. In the case of multiple modulation signals, the signal modulation frequencies can be optimized by minimizing the summation of their square for noise reduction.

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